LESSON 4.4d

Real Life Polynomial Based Problems

Today you will:

- Practice the mechanics of what you've been learning
- Apply what you've been learning to practical problems
- Practice using English to describe math processes and equations

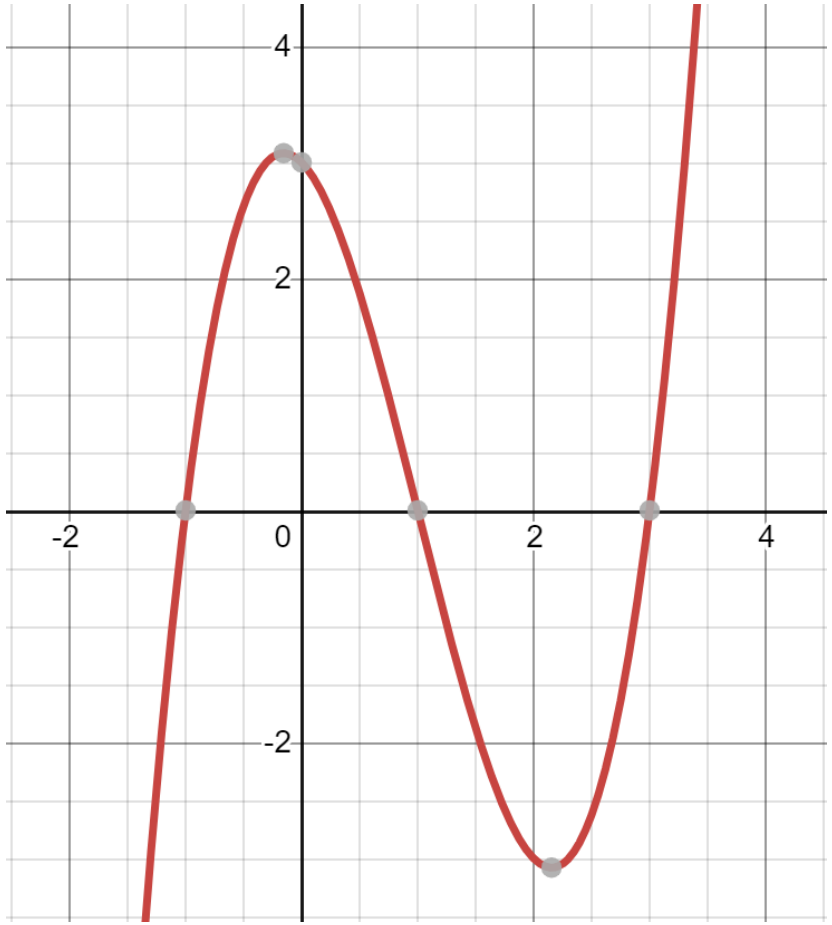
Given $f(x) = x^3 - 3x^2 - x + 3$:

1. Find the roots of $f(x)$ using your graphing calculator:

   …it appears the roots are -1, 1, 3 (verified using the table feature)

2. Using grouping, completely factor $f(x)$:

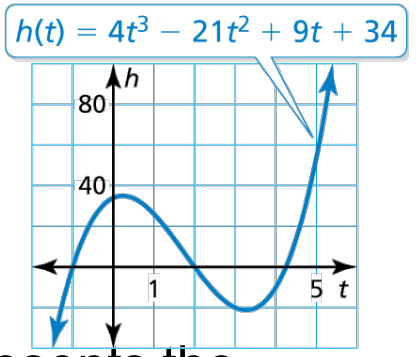
$$\begin{aligned} f(x) &= x^3 - 3x^2 - x + 3 \\ &= x^2(x-3) - (x-3) \\ &= (x^2-1)(x-3) \\ &= (x-1)(x+1)(x-3) \end{aligned}$$



Using the above information, what is the relationship between the factors of a polynomial function (when completely factored) and its roots?

- When completely factored, the polynomial factors give you the roots.
- Or looking at it the other way, if you know the roots, you can write the factors.

During the first 5 seconds of a roller coaster ride, the function $h(t) = 4t^3 - 21t^2 + 9t + 34$ represents the height h (in feet) of the roller coaster after t seconds. How long is the roller coaster at or below ground level in the first 5 seconds?



SOLUTION

Understand the Problem You are given a function rule that represents the height of a roller coaster. You are asked to determine how long the roller coaster is at or below ground during the first 5 seconds of the ride.

Make a Plan Use your graphing calculator to estimate the zeros of the function and check using the Factor Theorem. Then use the zeros to describe where the graph lies below the t-axis.

Solve the Problem From the graph, the zeros appear to be −1, 2 & between 4 - 5.

1. Use synthetic division to verify -1 is a zero ***AND*** factor the original function into two factors: (x+1) and a 2nd factor.
2. Then further factor the 2nd factor so you have the 3rd factor.
3. The 3rd factor will tell us exactly where the 3rd zero is (between 4 and 5).

We have now factored $h(t)$ into $(t+1)(4t^2 - 25t + 34)$ & shown -1 is a zero.

Use the 2nd factor to verify that 2 is also a zero. This will also give us the 3rd and final factor/root.

Step 1 Verify −1 is a zero using synthetic division and factor into two parts.

−1	4	−21	9	34
		−4	25	−34
	4	−25	34	0

$h(-1) = 0$, so -1 is a zero of h and $t + 1$ is a factor of $h(t)$.

STUDY TIP
You could also check that 2 is a zero using the original function, but using the quotient polynomial helps you find the remaining factor.

Step 2 Using the factor $(4t^2 - 25t + 34)$ and synthetic division, verify 2 is a zero. This will also give us the 3rd factor.

2	4	−25	34
		8	−34
	4	−17	0

← The remainder is 0, so $t - 2$ is a factor of $h(t)$ and 2 is a zero of h.

So, $h(t) = (t + 1)(t - 2)(4t - 17)$.

The 3rd factor $4t - 17$ indicates that the zero between 4 and 5 is $\frac{17}{4}$, or 4.25.

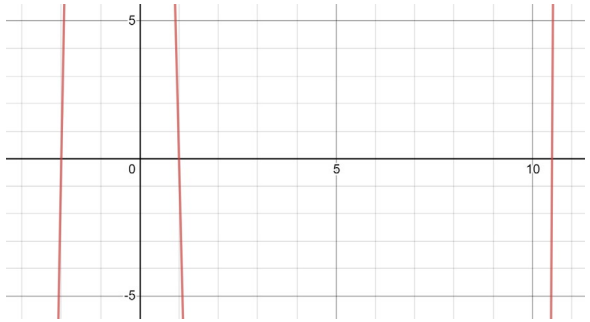
The zeros are −1, 2, and 4.25. Only $t = 2$ and $t = 4.25$ occur in the first 5 seconds. The graph shows that the roller coaster is at or below ground level for $4.25 - 2 = 2.25$ seconds.

4. Look Back Use a table of values to verify the positive zeros and heights between the zeros.

X	Y1
.5	33.75
1.25	20.25
2	0
2.75	-16.88
3.5	-20.25
4.25	0
5	54

X=2

During the first 12 seconds of a roller coaster ride, the function $g(t) = 2t^3 - 19t^2 - 25t + 42$ represents the height g (in feet) of the roller coaster after t seconds. How long is the roller coaster at or below ground level in the first 12 seconds?



SOLUTION

1. **Understand the Problem** You are given a function rule that represents the height of a roller coaster. You are asked to determine how long the roller coaster is at or below ground during the first 12 seconds of the ride.
2. **Make a Plan** Use your graphing calculator to estimate the zeros of the function and check using the Factor Theorem. Then use the zeros to describe where the graph lies below the t-axis.
3. **Solve the Problem** From the graph, two of the zeros appear to be -2 and 1. The third zero is between 10 and 11.

Step 1 Determine whether -2 is a zero using synthetic division.

-2	2	-19	-25	42
		-4	46	-42
	2	-23	21	0

← $g(-2) = 0$, so -1 is a zero of h and $t + 2$ is a factor of $g(t)$.

We have now factored $g(t)$ into $(t + 2)(2t^2 - 23t + 21)$ and shown -2 is a zero. Use the 2nd factor to verify that 1 is also a zero. This will also give us the 3rd and final factor/root.

Step 2 Using the factor $(2t^2 - 23t + 21)$ and synthetic division, verify 1 is a zero. This will give us our 3rd factor.

1	2	−23	21
		2	−21
	2	−21	0

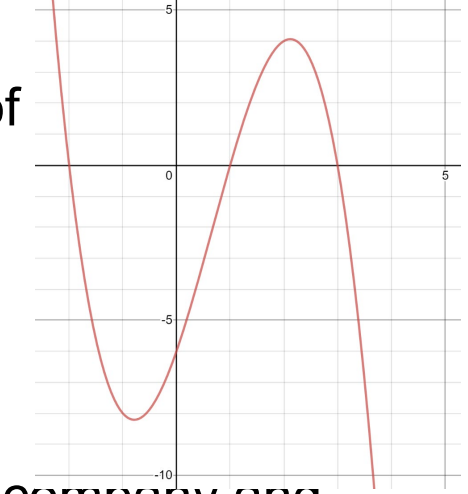
← The remainder is 0, so $t - 1$ is a factor of $h(t)$ and 1 is a zero of h.

So, $g(t) = (t + 2)(t - 1)(2t - 21)$.

The factor $2t - 21$ indicates that the zero between 10 and 11 is $\frac{21}{2}$, or 10.5.

The zeros are −2, 1, and 10.5. Only $t = 1$ and $t = 10.5$ occur in the first 12 seconds. The graph shows that the roller coaster is at or below ground level for $10.5 - 1 = 9.5$ seconds.

The profit P (in millions of dollars) for a hat manufacturer can be modeled by $P = -x^3 + 2x^2 + 5x$, where x is the number (in millions) of hats produced. The company currently produces 3 million hats and makes a profit of \$6 million, but it would like to cut back production. What lesser number of hats could the company produce and still make the same profit?



SOLUTION

1. **Understand the Problem**. You are given a function modeling the sales for a hat company and profit information. You need to determine a lower value for x that still yields the same profit.

2. **Make a Plan.** Create an equation for this situation by setting P equal to the desired profit.
   $-x^3 + 2x^2 + 5x = 6 \rightarrow -x^3 + 2x^2 + 5x - 6 = 0$
   Then find the roots … this will give you the values that produce the desired profit.

3. **Solve the Problem** Using your graphing calculator, the zeros appear to be -2, 1 and 3.
   Use this information to completely factor the function.
   This will give us the exact roots which we can use to answer this problem.

Step 1 Verify 2 is a zero using synthetic division (this factors the function into 2 factors).

-2	-1	2	5	-6
		2	-8	6
	-1	4	-3	0

$x + 2$ is a factor of P.

- We have now factored P into $(x + 2)(-x^2 + 4x - 3)$.
- We know we have three roots so factor $(-x^2 + 4x - 3)$.

Step 2 Factor ($-x^2 + 4x - 3$). You can use any method you know to factor this.
This is a quadratic so first see if you can directly factor it. If not, try the quadratic formula.

$$-x^2 + 4x - 3 =$$

Factor out the negative so we have a positive leading coefficient

$$= -(x - 3)(x - 1)$$

Directly factor the quadratic

So, $P = -(x + 2)(x - 3)(x - 1)$.

- The zeros are -2, 1, and 3. Again, x is the number of hats produced and here the zeros represent the values of x for which the profit is \$6 million.

  The manufacturer currently produces 3 million hats.

  Our work says that if the manufacturer produces 1 million hats they will realize the same profit of \$6 million!

Homework

Pg 185, #56-67